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OPTICAL-MECHANICAL ANALOGY

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16. Abstract

The authors consider the propagation of an electromagnetic wave in a spherically refracting medium on the basis of the optical-mechanical analogy between the trajectory of the ray and the trajectory of a material particle and use it to determine the refractive index of the medium. As an example of an application, the authors consider the refraction of radio waves in the solar corona.

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OPTICAL-MECHANICAL ANALOGY

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In the study of the refraction of electromagnetic waves, propagating in a spherically refracting medium, in the optical-geometric approximation, the apparatus developed for the motion of a material point (particle) in the central field can be used on the basis of the well-known analogy between the ray and the trajectory of the particle [1]. /3*

According to the Fermat principle, for electromagnetic rays propagating in a refracting medium

$$\delta \int \frac{ds}{v_\phi} = \frac{1}{c}\,\delta \int n\,ds = 0, \tag{1}$$

where v_ϕ is the phase velocity of the wave, n is the refractive index, c is the velocity of light in vacuum and ds is an element of length of the ray.

On the other hand, for a particle moving in a potential field U with constant energy E, according to the Maupertu principle

$$\delta \int mv\,ds = \delta \int \sqrt{2m(E-U)}\,ds = 0, \tag{2}$$

where m is the mass of the particle, v is the velocity, $E = 1/2\ mv^2 + U = \text{const}$ and ds is an element of the length of the particle trajectory.

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* Numbers in the margin indicate pagination in the foreign text.

Henceforth, we will consider everywhere the trajectory of a particle in a fixed central field U(r) and the trajectory of a ray in a spherically refracting medium, i.e. n = n(r). /4

By comparing (1) and (2), it can be seen that the trajectory of the ray coincides with the trajectory of the particle when

$$n^2(r) = 1 - \frac{U(r)}{E} \tag{3}$$

or

$$n(r) = \frac{v(r)}{c} .$$

In analogy with the law for the conservation of momentum of the particle

$$m v r \sin\varphi = M = const \tag{4}$$

we can obtain from (1) the relation

$$n r \sin\varphi = p = const \tag{5}$$

where ϕ is the angle between the radius vector $\vec{r}$ and the tangent to the trajectory (ray). Equation (5) is usually called the equation of the ray in a spherically refracting medium. Using (5), in polar coordinates r, θ, the equation of the ray can be expressed in the following form

$$\theta = \int \frac{\frac{p}{r^2}\, dr}{\sqrt{n^2 - \frac{p^2}{r^2}}} + const \tag{6}$$

Equation (6) is analogous to the equation for the trajectory of the particle in polar coordinates [2]:

$$\theta = \int \frac{\frac{M}{r^2}\, dr}{\sqrt{2m[E - U(r)] - \frac{M^2}{r^2}}} \tag{7}$$

The values of r for which the denominator in (6) vanishes, i.e. /5

$$rn(r) = p \tag{8}$$

determine the boundaries of the region in which the ray propagates. When the region of admissible values r is bounded only by one condition $r \geq r_{min}$, the propagation of the ray is infinite. This corresponds to the case when the receiver and emitter are located at $r \geq r_{min}$. When the region over which r varies has two boundaries r_{min} and r_{max}, the trajectory of the ray lies entirely in the interior of the ring

$$r_{min} < r < r_{max}.$$

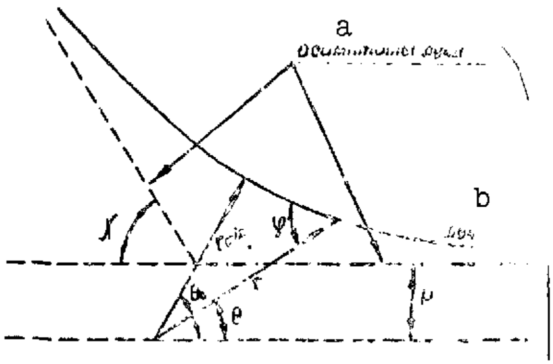
2. Determination of the Refractive Index of a Spherically Refracting Medium from the Total Refraction Angle

We will consider the propagation of electromagnetic waves in a spherically refracting medium, when the region in which the ray propagates is bounded by the condition $r > r_{min}$.

In polar coordinates r, θ, where θ is measured in the direction of the point r_{min} where the ray is deflected, the equation of the ray (6) takes on the following form

$$\theta(r, p) = \pm \int_{r_{min}}^{r} \frac{p\, dr}{r\sqrt{(rn)^2 - p^2}} \tag{9}$$

where p is the impact parameter of the ray (see figure).



Refraction geometry of ray.
Key: a. Ray asymptotes
b. Ray

When the receiver and emitter are far behind the boundaries of the refracting medium, it can be assumed that they lie on the ray asymptotes. /6

The angle θ_0 is

$$\theta_0(p)=\int_{r_{min}}^{\infty}\frac{p\,dr}{r\sqrt{(rn)^2-p^2}} \tag{10}$$

The total refraction angle χ is

$$\chi(p)=\pi-2\theta_0. \tag{11}$$

(10) and (11) imply

$$\chi(p)=\pi-2\int_{r_{min}}^{\infty}\frac{p\,dr}{r\sqrt{(rn)^2-p^2}} \tag{12}$$

Let us assume that it was possible to measure experimentally the relation $\chi(p)$ for $p > p_0$. Then, assuming that $1 - n(r)$ is a monotonically decreasing function, we can reconstruct the relation $n(r)$. To do this, taking advantage of the optical- /7 mechanical analogy, we can use the known solution of the nonlinear integral equation (12) for $n(r)$ (see [2], §18, problem 7). The final result can be expressed in the following form

$$n(r)=\exp\left\{-\frac{1}{\pi}\int_{rn}^{\infty}\operatorname{Arch}\frac{p}{rn}\cdot\frac{d\chi}{dp}\,dp\right\}=\exp\left\{\frac{1}{\pi}\int_{rn}^{\infty}\frac{\chi(p)\,dp}{\sqrt{p^2-r^2n^2}}\right\}. \tag{13}$$

Formula (13) implicitly defines the relation for all

$$r > r_{min} = \frac{p}{n(r_{min})}$$

Let us consider the case of a weakly refracting medium, i.e.

$$n(r) = 1 - \Delta n(r) ,$$

where $\Delta n(r) \ll 1$.

Under this assumption, the expression (12) for $\chi(p)$ can be simplified by analogy with problems 1 and 2, §20 [2]. Using a series expansion in powers of Δn and transforming expression (12), it is possible to obtain:

$$\chi(p) = -2p \int_p^\infty \frac{d\Delta n(r)}{dr} \cdot \frac{dr}{\sqrt{r^2 - p^2}} \tag{14}$$

Using the well-known solution of Abel's integral equation, we can write

$$\Delta n(r) = -\frac{1}{\pi} \int_r^\infty \frac{\chi(p)\, dp}{\sqrt{p^2 - r^2}}$$

An expression for $\chi(p)$ which is similar to (14) was derived in a somewhat different manner in [3].

Let us consider the case when Δn is small and depends in the following manner on r /8

$$\Delta n(r) = \sum_{m \geq 2} \alpha_m / r^m , \tag{15}$$

where m and α_m are constants. According to (14)

$$\chi(p) = 2p \sum m \alpha_m \cdot \int_p^\infty \frac{dr}{r^{m+1}\sqrt{r^2 - p^2}} \tag{16}$$

Using the substitution $p^2/r^2 = u$, integral (16) can be reduced to an Euler β-integral, and it can be expressed in terms of gamma functions

$$\chi(p) = 2 \sum_{m=2} \frac{\sqrt{\pi}}{p^m} \cdot \alpha_m \cdot \frac{\Gamma\left(\frac{m+1}{2}\right)}{\Gamma\left(\frac{m}{2}\right)} \tag{17}$$

3. Application to the Refraction of Radio Waves in the Regular Spherically Symmetric Solar Corona

It is known that in the transillumination of the solar corona by radio waves in the meter range in the region $p < 10R_o$ (see figure, $R_o \simeq 6.96 \cdot 10^{10}$ cm is the radius of the photosphere), it is necessary to take into account, in addition to the scattering of the electron concentration by the inhomogeneities, also the refraction of the radio waves in the corona [4].

For radio waves with wavelength $\lambda = c/f \leq 8$ m (f is the wave frequency and c the velocity of light), we can assume that $\Delta n(r) \ll 1$ in the solar corona in the region of heliocentric distances $r \gtrsim 3$. Here

$$\Delta n = 1 - n = \frac{2\pi e^2 N}{m(2\pi f)^2} \simeq \frac{4.03 \cdot 10^7}{f^2} N \tag{18}$$

where m is the mass of the electron, N is the electron concentration in el/cm^3 and e is the electron charge.

To estimate the refraction angle of the radio waves in the corona, we will assume that some average distribution of the electron concentration in the corona has the following form: /9

$$N(\eta) = 10^{8}(3\eta^{-16} + 1,5\eta^{-6}) + 4,58\cdot 10^{4} n_e \eta^{-2}, \tag{19}$$

where $\eta = r/R_0$ and n_e is the numerical value of the electron concentration on the Earth's orbit ($\eta \sim 214$). The first two terms in the right member of (19) are known as the Allen-Baumbach formula. The additional term in (19) describes the results of the measurements on the Earth's orbit. Substituting (19) in (18), we obtain

$$\Delta n = \alpha/\eta^{6} + \beta/\eta^{2} \quad ,$$

where α = const and β = const for a fixed value of f. The term $\sim\eta^{-16}$ in (19) can be ignored, since it describes the distribution of the average electron concentration in the region $1.03 < \eta < 1.05$. Using (17), we can obtain the relation $\chi(p)$ in analytical form

$$\chi(p) = A p^{-6} + B p^{-2},$$

where

$$A \simeq 0,35\cdot 10^{17}\cdot \frac{1}{f^2}, \quad B \simeq 0,57\cdot 10^{13}\cdot n_e \frac{1}{f^2}.$$

The table gives the relation $\chi(p)$ for waves with $f = 10^8$ Hz (λ_0 = 3 m) for n_e = 6,10,30.

$\rho / R_\odot$	χ' $n_e = 6$	χ' $n_e = 10$	χ' $n_e = 30$
3	17.7′	18.4′	22.8′
4	3.7′	4.1′	6.6′
6	0.6′	0.8′	1.9′
8	0.23′	0.35′	0.95′
10	0.13′	0.21′	0.60′

It can be seen from the table that for the model of the average electron concentration in the corona that was adopted, the total refraction angle χ is not large. /10

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